Mathematical Modelling Of High-Frequency Field in Structurally-Heterogeneous Medium

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Abstract: **This paper investigated the effective parameters of composite materials in the case of external stress with finite length of electromagnetic (EM) - waves in the presence of essential gradients and frequency characteristics of the field. During the propagation of long waves, the approximation of electromagnetic field can be characterized by dielectric and magnetic permeability, the value of which is determined by comparing the main characteristics of the field in anisotropic, homogeneous and heterogeneous mediums. During the propagation of short waves, in the obtained solution, propagation effects in the structure are taken into account, as effective parameters strongly depend on the frequency of the field.**

Keywords: **heterogeneous medium, high-frequency field, electromagnetic waves, mathematical modeling, radio polarization method.**

I. INTRODUCTION

The reliability of a construction significantly depends on the location of technological defects such as cracks, voids, bundles and external influences in structural elements. Therefore, it is necessary to develop methods for finding defects by means of nondestructive testing (NDT), followed by an analysis of their influence on the strength of the structure. In this case, it is most effective to employ the microwave range (MR) of electromagnetic waves. Developments of effective methods (NDT) are largely associated with mathematical modeling of NDT. It is known that the electromagnetic properties of composite medium in the long-wave approximation are determined by effective permittivity, permeability, and conductivity and so on [7]. The macrostructure of a field gives important information about violations of the constant structure of the composite and can be studied by the radiopolarization (NDT) – method[3,5]. Fundamental research in mathematical modeling of dispersion of EM - waves on heterogeneities has been carried out by L.M. Brekhovskiy [1], G.A. Vanin [6], B.I. Kolodiy [2], A.G. Ramm [4].

II. FORMULATION AND MATHEMATICAL MODEL OF THE PROBLE

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Let us consider the medium in which the fibers have coherent orientation. Let the distance between the centers of adjacent fibers be d_1, d_2 ; α be an angle grid . We can determine the coordinates of the center mn-th fiber from this relation
$x_2 + ix_3 = d(m+bne^{i\alpha}),\ m,n = 0,\pm 1,\dots\dots,\pm\infty.$

where $d_1\ be^{i\alpha} = d_2,$ and Ox -axis coincides with the axis of the fiber. Maxwell's equations for electromagnetic wave propagation in isotropic medium will have the form

$$rot\,\bar{H} = \frac{1}{c}\frac{\partial \bar{D}}{\partial t} + \frac{4\Pi}{c}\bar{J},\quad rot\,\bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t}$$

$$\bar{D} = \varepsilon\bar{E},\quad \bar{B} = \mu\bar{H},\quad \bar{j} = \sigma\bar{E} \tag{1}$$

where $\bar{j}$ is voltage vector , c is speed of light; σ is conductivity. $\bar{E}$ and $\bar{H}$ are voltage vectors of electric and magnetic fields, ε and μ are dielectric and magnetic permeability of the medium. For dielectrics at given frequencies , changes of the field equals σ = 0, j = 0. The solution of equation (1) for stationary wave propagation is as follows;

$$\bar{H} = \bar{H}_0 e^{-iwt} \quad , \quad \bar{E} = \bar{E}_0 e^{-iwt}$$

$$\bar{D} = \bar{D}_0 {}^{-iwt} , \quad \bar{B} = \bar{B}_0 e^{-iwt}$$

The general solution for (1) was found as a sum of electric - $\bar{\pi}$ and magnetic - $\bar{\pi}^*$ of Hertz vectors for each component of the medium.

$$\bar{E} = \kappa\varepsilon\mu\bar{\pi} + grad \operatorname{div}\bar{\pi} + i\kappa\mu \operatorname{rot}\bar{\pi}^*,$$

$$\bar{H} = -ik\varepsilon \operatorname{rot}\bar{\pi} + k^2\varepsilon\mu\bar{\pi}^* + grad \operatorname{div}\bar{\pi}^*, \qquad (2)$$

where $\kappa = w/c$; $\bar{\pi}$ and $\bar{\pi}^*$ satisfy the same equation

$$\frac{\partial^2 \bar{\pi}}{\partial x_1^2} + \frac{\partial^2 \bar{\pi}}{\partial x_2^2} + \frac{\partial^2 \bar{\pi}}{\partial x_3^2} + \kappa^2\varepsilon\mu\bar{\pi} = 0$$

Boundary condition reduces to the continuity on the surfaces of contact fibers with the matrix of tangential voltage vector components of the electric and magnetic fields.

$$n \times \bar{E}\Big|_{\Omega mn} = n \times \bar{E}\big|_{\Omega mn} , \quad n \times \bar{H}\Big|_{\Omega mn} = n \times \bar{H}\big|_{\Omega mn} \qquad (3)$$

$m,n = 0 \pm 1, \ldots, \pm \infty$

where n is normal vector to the cylindrical surface of the fiber. One component of Hertz vector is maintained in the cylindrical coordinate system. Here and further in the article $\bar{e}_1$, $\bar{e}_r$, $\bar{e}_\theta$ are unit vectors of locally cylindrical coordinate system.

In the case of transverse magnetic wave, electromagnetic field in a local cylindrical coordinate system, in accordance with (2) is presented with the following

$$\bar{E} = \bar{e}_1\left(k^2\varepsilon\mu\pi + \frac{\partial^2\pi}{\partial x_1^2}\right) + \bar{e}_r \frac{\partial^2\pi}{\partial r \partial x_1} + \bar{e}_\theta \frac{1}{r}\frac{\partial^2\pi}{\partial\theta\partial x_1} ; \qquad (4)$$

$$\bar{H} = ik\varepsilon \bar{e}_r \frac{1}{r}\frac{\partial\pi}{\partial\pi} + ik\varepsilon \bar{e}_\theta \frac{\partial\pi}{\partial r} .$$

For transverse - electric field we have these equations

$$\bar{E} = ik\mu \bar{e}_r \frac{1}{r}\frac{\partial\pi^*}{\partial\theta} - ik\mu \bar{e}_\theta \frac{\partial\pi}{\partial r} ,$$

$$\bar{H}^* = \bar{e}_1\left(k^2\varepsilon\mu\pi^*\right) + \bar{e}_r \frac{\partial^2\pi^*}{\partial r\partial x_1} + \bar{e}_\theta \frac{1}{r}\frac{\partial^2\pi^*}{\partial r\partial x_1} + \bar{e}_\theta \frac{1}{r}\frac{\partial^2\pi^*}{\partial\theta\partial x_1} \qquad (5)$$

Depending on concepts of common field, boundary conditions are written as follows:

$$\bar{E}_1\Big|_{\Omega mn} = E'_{1a}\big|_{\Omega mn} , \quad E'_\theta + E^*_\theta\big|_{\Omega mn} = E'_{\theta\alpha} + E^*_{\theta\alpha}\big|_{\Omega mn}$$

$$H^*_1\big|_{\Omega mn} = H^*_{1a} , \quad H'_\theta + H^*_\theta\big|_{\Omega mn} = H'_{\theta\alpha} + H^*_{\theta\alpha}\big|_{\Omega mn}$$

where $\bar{E}_a = \varepsilon_{1a}\bar{e}_1 + E_a\bar{e}_0 + E_{r0}\bar{e}_r$ and e.t.c.

In general, the electromagnetic field in the matrix is the sum of functions

$$\pi' = \sum_{\tau=-\infty}^{\infty} e^{i\tau v_{mn} + ihx_1} \left\{ v_{mn}^{\tau} j_\tau(\lambda r_{mn}) + B_{mn}^{\tau} j_\tau(\lambda r_0) H_\tau^{(1)}(\lambda r_{mn}) \right\},$$

$$\pi^* = \sum_{\tau=-\infty}^{\infty} e^{i\tau\theta_{mn} + ihx_1} \left\{ W_{mn}^{\tau} j_\tau(\lambda_{r_{mn}}) + D_{mn}^{\tau} j_\tau(\lambda r_0) H_\tau^{(1)}(\lambda_{mn}) \right\}$$

The wave field in the domain, occupied by mn – fiber, is found in the rows form by standing waves

$$\pi_a = \sum_{mn}^{\tau} j_\tau(\lambda_s r_0) j_\tau(\lambda_a r_{mn}) e^{i\tau v_{mn} + ihx_1},$$

$$\pi_a^* = \sum_{\tau=-\infty}^{\infty} c_{mn}^{\tau} j_\tau(\lambda_s r_0) j_\tau(\lambda_a r_{mn}) e^{i\tau v_{mn} + ihx_1}$$

$$\pi_a^* = \sum_{\tau=-\infty}^{\infty} c_{mn}^{\tau} j_\tau(\lambda_s r_0) j_\tau(\lambda_a r_{mn}) e^{i\tau v_{mn} + ihx_1}$$

Constants A,B,C,D can be establish from the boundary conditions in the following forms

$$\left(\kappa^2 \varepsilon\mu + \frac{\partial^2}{\partial x_1^2}\right)\pi\Big|_{r=r_0} = \left(\kappa^2 \varepsilon_a \mu_a + \frac{\partial^2}{\partial x_1^2}\right)\pi_a\Big|_{r=r_0},$$

$$\frac{1}{r_0}\frac{\partial^2 \pi}{\partial v \partial x_1} - i\kappa\mu\frac{\partial \pi^*}{\partial r}\Big|_{r=r_0} = \frac{1}{r_0}\frac{\partial^2 \pi_a}{\partial v x_1} - ik\mu_a\frac{\partial \pi_a}{\partial r}\Big|_{r=r_0},$$

$$\left(\kappa^2 \varepsilon\mu + \frac{\partial^2}{\partial x_1^2}\right)\pi^*\Big|_{r=r_0} = \left(\kappa^2 \varepsilon_a \mu_a + \frac{\partial^2}{\partial x_1^2}\right)\pi_a^*\Big|_{r=r_0},$$

$$i\kappa\varepsilon\frac{\partial \pi}{\partial r} + \frac{1}{r}\frac{\partial^2 \pi}{\partial v \partial x_1}\Big|_{r=r_0} = i\kappa\varepsilon_a\frac{\partial \pi}{\partial r} + \frac{1}{r}\frac{\partial^2 \pi_a^*}{\partial v \partial x_1}\Big|_{r=r_0}$$

The functions of the electromagnetic field after unwieldy transformation can be written in the form

$$\pi = \sum_{\tau=-\infty}^{\infty} \left\{ v_{mn}^{\tau}\left[j_\tau(\lambda r_{mn}) - z_\tau H_\tau^{(1)}(\lambda r_{mn})\right] - W_{mn}^{\tau} Y_\tau^* H_\tau^{(1)}(\lambda r_{mn}) \right\} e^{i\tau v_{mn} + hx_1}, \quad (6)$$

$$\pi^* = \left\{ W_{mn}^{\tau}\left[j_\tau(\ r_{mn}\ )\right] - Y_\tau H_\tau^{(1)}(\lambda r_{mn})\ \right] - V_{mn}^{\tau} Z_\tau^{(1)}(\lambda r_{mn}) \right\} e^{i\tau v_{mn} + ihx_1},$$

$$\pi_a = \frac{\lambda_s}{\lambda_a}\sum_{\tau=-\infty}^{\infty} \left\{ V_{mn}^{\tau}\left[j_\tau(\lambda r_0) - Z_\tau H_\tau^{(1)}(\lambda r_0)\ \right] - W_{mn}^{\tau} Y_\tau^* H_\tau^{(1)}(\lambda r_0) \right\} \frac{j_\tau(\lambda_a r_{mn})}{j_\tau(\lambda_a r_0)} e^{i\tau v_{mn} + ihx_1},$$

$$\pi_a^* = \frac{\lambda_s}{\lambda_a}\sum_{\tau=-\infty}^{\infty} \left\{ W_{mn}\left[j_\tau(\lambda r_0) - Y_\tau H_\tau^{(1)}(\lambda r_0)\ \right] - V_{mn}^{\tau} Z_\tau^* H_\tau^{(1)}(\lambda r_0) \right\} \frac{j_\tau(\lambda_a r_{mn})}{j_\tau(\lambda_a r_0)} e^{i\tau v_{mn} + ihx_1},$$

where

$$Z_\tau = -L_\tau^{-1}\left\{ h^2\tau^2 r_0^2 j_\tau(\lambda r_0)\left(1 - \lambda^2/\lambda_a^2\right) - \lambda^2 r_0^4 k^2 \varepsilon\mu j_\tau(\lambda r_0)\left[\frac{H_\tau'(\tau r_0)}{H_\tau(\lambda r_0)} - \frac{\lambda\mu_a}{\lambda_a\mu}\frac{j_\tau'(\lambda_a r_0)}{j_\tau(\lambda_a r_0)}\ \right]\left[\frac{j_\tau'(\lambda r_0)}{j_\tau(\lambda r_0)} - \frac{\lambda\varepsilon_a}{\lambda_a\varepsilon}\frac{j_\tau'(\lambda_a r_0)}{j_\tau(\lambda_a r_0)}\right] \right\},$$

$$L_\tau = \lambda_s^2 r_0 k^2 \varepsilon\mu\left[\frac{H_\tau'(\lambda r_0)}{H_\tau(\lambda r_0)} - \frac{\lambda\mu_a}{\lambda_a\mu}\frac{j_\tau'(\lambda r_0)}{j_\tau(\lambda r_0)}\right]\left[H_\tau'(\lambda r_0) - \frac{\lambda_\varepsilon}{\lambda_a\varepsilon}H(\lambda r_0)\frac{j_\tau'(\lambda_a r_0)}{j_\tau(\lambda_a r_0)}\right] - r_0^2 h^2\tau^2\left(1 - \frac{\lambda_s^2}{\lambda_a^2}\right)^2 H_\tau(\lambda r_0),$$

$$Y_\tau^* = -L_\tau^{-1} i\left(1 - \frac{\lambda^2}{\lambda_a^2}\right) r_0^3 h\tau k\mu\lambda j_\tau(\lambda r_0)\left[\frac{j_\tau'(\lambda r_0)}{j_\tau(\lambda) r_0} - \frac{H_\tau'(\lambda r_0)}{H_\tau(\lambda r_0)}\right], \quad (7)$$

$$Z_\tau^* = -L_\tau^{-1} i\left(1-\frac{\lambda^2}{\lambda_a^2}\right) r_0^3 h\tau\mu j_\tau(\lambda r_0)\left[\frac{H'_\tau(\lambda r_0)}{H_\tau(\lambda r_0)} - \frac{j'_\tau(\lambda r_0)}{j_\tau(\lambda r_0)}\right],$$

$$Y = -L_\tau^{-1}\left\{ r_0^2 h^2\tau^2\left(1-\frac{\lambda^2}{\lambda_a^2}\right)^2 j_\tau(\lambda r_0) - r_0^4 k^2\varepsilon\mu\lambda^2 j_\tau(\lambda r_0)\left[\frac{H'_\tau(\lambda_s r_0)}{H_\tau(\lambda_s r_0)} - \frac{\lambda\varepsilon_a}{\lambda_a\varepsilon}\frac{j'_\tau(\lambda_a r_0)}{j_\tau(\lambda_a r_0)}\right]\left[\frac{j'_\tau(\lambda r_0)}{j_\tau(\lambda r_0)} - \frac{\lambda\mu_a}{\lambda_a\mu}\frac{j'_\tau(\lambda_a r_0)}{\lambda_a\mu j_\tau(\lambda_a r_0)}\right]\right\}$$

.

In formulas (7), and in others, superscript in Hankel function is omitted.

Falling on the surface Ω, wave field is the sum of the reflected waves from all fibers. By adding the reflected waves from surrounding inhomogeneities and using theorem of combining cylindrical functions we can find the following system of equations

$$V_{mn}^s + \sum_{p\neq m,}\sum_{q\neq n}\sum_{\tau}\left[Z_\tau V_{pq}^\tau + Y_{pq}^*\right] H_{s+\tau}\left(\lambda R_{pq}^{mn}\right) e^{-isv_{pq}^{mn}} = 0 \quad , \qquad (8)$$

$$V_{mn}^s + \sum_{p\neq m}\sum_{q\neq n,}\sum_{\tau}\left[Y_\tau W_{pq}^\tau + Z_q^* V_{pq}^\tau\right] H_{s+\tau}\left(\lambda R_{pq}^{mn}\right) e^{-isv_{pq}^{mn}} = 0$$

Here subscripts *mn* and *pq* coincide with number of fibers cell in a Cartesian coordinate system;

$$R_{p\rho}^{mn} = a_1\sqrt{(p-m)^2 + 2b(p-m)(q-n)\cos\alpha + b^2(q-n)^2}\ ;$$

V_{pq}^{mn} is an angle subtended at the center of pq-th fiber from mn -th . The wave field consists of constructing functions V_{mn}^s and W_{mn}^s, determining the field in some cells, and functions that characterize the field in a separate cell.

Further in the article, and for the convenience of definitions of the characteristics of the electromagnetic field and the analysis of the solutions, waves that are propagating transversely and longitudinally with respect to the orientation of the fibers are studied separately.

Let us consider the case of transverse wave propagation. Assuming h = 0, solution of the system (8) is found in the form of traveling transverse waves.

$$V_{mn}^s = C_s \exp\{iad_1[m\cos\varphi + bn\cos(\alpha-\varphi)]\}, \qquad (9)$$

$$W_{mn}^s = S_s \exp\{iad_1[m\cos\varphi + bn\cos(\alpha-\varphi)]\},$$

where φ is an angle between the direction of wave propagation and the positive direction of the axis– *Ox*. Using these conversions

$$\rho e^{iv_{p\rho}^{mn}} = p - m + b(\rho - n)e^{i\alpha}, \quad V_{p\rho}^{mn} = \varphi + \varphi + \frac{\pi}{2}$$

we can introduce polar system of reference points (8). When substituting the solution (9) into (8) we obtain the system of algebraic equations

$$\sum_\tau\left[(\delta_{s\tau} + Z_\tau G_{s\tau})C_\tau + Y_\tau^* G S_\tau\right] = 0$$

$$\sum_\tau\left[(\delta_{s\tau} + Y_\tau G_{s\tau}) S_\tau + Z_\tau^* G C_\tau\right] = 0, \qquad s,\tau = 0,\pm1,\ldots,\pm\infty$$

,

which follows from these equations that infinite determinant equal to zero

$$\begin{Vmatrix} \delta_{s\tau} + Z_\tau G_{s\tau} & Y_\tau^* G_{s\tau} \\ Z_\tau^* G_{s\tau} & \delta_{s\tau} + Y_\tau G_{s\tau} \end{Vmatrix} = 0 \qquad s,\tau = 0,\pm1,\ldots,\pm\infty \qquad (10)$$

where $\delta_{s\tau}$ is Kronecker symbol:

$$G_{s\tau} = i^s e^{-is\varphi}\sum_\psi\sum_\rho{}' H_{s+\tau}(d_1\lambda_\rho) e^{ia\rho d_1\sin\psi - is\psi} \qquad (11)$$

The prime mark means that the term is excluded from the sum at $\rho = 0$, then $a_j - j -$ th root of infinite determinant (10), then relationship between the variables are established

$$C_s^j = g_s(a_j)C_0^j, \qquad S_s^j = g_s'(a_j)S_0^j$$

Generally, functions of the electromagnetic field have solutions which corresponds to all the following j:

$$\pi_s = \sum_j \exp\{ia_j d_1[m\cos\varphi + b\,n\cos(a-\varphi)]\times$$
$$\left\{C_j \sum_{r=-\infty}^{\infty} g_r(a_j)[S_\tau(\lambda r_{mn}) - Z_\tau H_\tau(\lambda r_{mn})]\, e^{i\tau v_{mn}} -\right.$$
$$\left. -S_j \sum_{r=-\infty}^{\infty} g'_\tau(a_j) Y_\tau^* H_\tau(\lambda r_{mn}) e^{i\tau v_{mn}}\right\},$$
$$\pi_s^* = \sum_j \exp\{ia_j d_1[m\cos\varphi + bn\cos(a-\varphi)]\}\times$$
$$\left\{S_j \sum_{r=-\infty}^{\infty} g_r'(a_j)\left[\int_\tau(\lambda r_{mn}) - Y_\tau H_\tau(\lambda r_{mn})\right] e^{i\tau v_{mn}} -\right.$$
$$\left. C_j \sum_{r=-\infty}^{\infty} g_\tau(a_j) Z_\tau^* H_\tau(\lambda r_{mn}) e^{i\tau v_{mn}}\right\} \tag{12}$$

We can also write the same ratio for functions π_a and π_a^* of the field. The vector components of the electric and magnetic fields are calculated directly with (12).

In the long wavelength approximation, when the field changes slightly in adjacent cells, the sum of (11) can be replaced by next integral

$$\sum_\psi \sum_\rho{}' H_{s+\tau}(d_1\lambda_\rho) e^{ia\rho d_1\sin\psi - is\psi} \cong \frac{1}{F}\int_\varepsilon^\infty \int_0^{2\pi} \rho d_\rho d_\psi H_{s+\tau}(\lambda\rho) e^{ia\rho\sin\psi - is\psi} \quad ,$$

where $F = bd_1^2\sin\alpha$ is the area of elements cells in the section ; using Poisson formula, relation (11) becomes

$$G_{s\tau} = \frac{2\pi i^s}{F} e^{-is\psi} \int_\varepsilon^\infty \rho d_\rho S_s(a\rho) H_{s+\tau}(\lambda\rho) \tag{13}$$

With slight changes in the field and convergence of the integral in the interval, lower limit will be $\delta = 0$. Integration by parts using Bessel equation, integral (13) becomes

$$\int_\delta^\infty \rho d\rho \int_\tau(a\rho) H_n(\lambda\rho) = \frac{n^2 - S^2}{\lambda^2 - a^2}\int_\delta^\infty \frac{d\rho}{\upsilon}\int_s(a\rho) H_n(\lambda\rho) +$$
$$+\frac{1}{\lambda^2 + a^2}\times\left[a\rho\int_{s-1}(a\rho)H_n(\lambda\rho) - \lambda\rho\int_s(a\rho)H_{n-1}(\lambda\rho) + (n-s)\int_s(a\rho)H_n(\lambda\rho)\right]_\delta^\infty \tag{14}$$

The supplement in brackets at the upper limit approaches to the sum of δ - functions of the argument $a \pm \lambda$. Excluding the above cases, further in the final result, these generalized functions are omitted. At the lower limit when using the asymptotic Bessel functions with a small argument we obtain $\frac{i\left(a/\lambda\right)^s}{\lambda^2 - a^2}\left(\frac{2}{\lambda\delta}\right)^{n-s}(n-s)\frac{\Gamma(n)}{\pi\Gamma(1+s)}$, here $\Gamma(n)$ - is the Gamma function.

The integral in (14) is expressed through discontinuous integrals of Weber - Shafheylina and similar to them. As a result, we can write:

$$\int_{\delta}^{\infty}\rho d\rho\int_{s}(a\rho)H_{n}(\lambda_{\rho})=\frac{n^{2}-s^{2}}{\lambda^{2}-a^{2}}\frac{\Gamma\left(\frac{n+s}{2}\right)}{2\Gamma(1+s)\Gamma\left(1+\frac{n-s}{2}\right)}\times$$

$$\times\begin{cases}\Lambda_{2}^{s}F_{1}\left(\frac{n+s}{2},\frac{s-n}{2};\quad 1+s;\Lambda^{2}\right)\\ \Lambda_{2}^{-s}F_{1}\left(\frac{n+s}{2},\frac{s-n}{2};\quad 1+s;\Lambda^{-2}\right)\end{cases}+\frac{i\Lambda^{\delta}}{\lambda^{2}-a^{2}}\left(\frac{2}{\lambda_{\varepsilon}}\right)^{n-s}(n+s)\frac{\Gamma(n)}{\pi\Gamma(1+s)}+ \tag{15}$$

$$+i\frac{n^{2}-s^{2}}{\lambda^{2}-a^{2}}\times\begin{cases}-\frac{\cos\frac{\pi(s-n)}{2}\Gamma\left(\frac{s+n}{2}\right)\Gamma\left(\frac{s-n}{2}\right)}{2\pi\Gamma(1+s)} & \Lambda_{2}^{s}F_{1}\left(\frac{s+n}{2},\frac{s-n}{2};1+s;\Lambda^{2}\right), \quad \Lambda\langle 1\\ \frac{\cos\frac{\pi(s-n)}{2}\Gamma\left(\frac{s+n}{2}\right)\Gamma\left(\frac{n-s}{2}\right)}{2\pi\Gamma(1+n)} & \Lambda_{2}^{-4}F_{1}\left(\frac{n+s}{2},\frac{n-s}{2};1+n;\Lambda^{-2}\right) \quad +\\ +\frac{\sin\frac{\pi(s+n)}{2}\Gamma\left(\frac{s+n}{2}\right)\Gamma\left(\frac{n-s}{2}\right)\Gamma\left(\frac{n-s}{2}\right)\Gamma\left(-\frac{n+s}{2}\right)}{2\pi^{2}\Gamma(\delta)} & \times\\ \times\Lambda_{2}^{-4}F_{1}\left(\frac{n+s}{2},\frac{n-s}{2};\varepsilon;1-\Lambda^{2}\right), & \Lambda\rangle 1\end{cases}$$

where $\Lambda=\frac{a}{\lambda}$; $2F_{1}(a,b;c;x^{2})$ is the Hypergeometric function,

$$2F_{1}(a,b;c;x^{2})=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{m=0}^{\infty}\frac{\Gamma(m+a)\Gamma(m+b)}{\Gamma(m+c)\Gamma(m+1)}\Lambda^{2m};$$

Similar simplification using asymtptote of cylindrical functions with a small argument are carried out with parameters

Z_{s}, S_{s}^{*} i Y_{s}, Y_{s}^{*}

$Y_{0}^{*}\cong 0,\quad Z_{0}^{*}=0,$

$Z_{-\tau}=Z_{\tau},\quad Y_{-\tau}=Y_{\tau},\quad Y_{-\tau}^{*}=Y_{\tau}^{*},\quad Z_{-\tau}^{*}=Z_{\tau}^{*}$

$$Z_{-\tau}\cong\frac{\pi i}{\Gamma(\tau)\Gamma(1+\tau)}\left(\frac{\lambda_{s}r_{0}}{2}\right)^{2\tau}\frac{\left(1-\frac{\varepsilon_{a}\lambda^{2}}{\varepsilon\lambda_{a}^{2}}\right)\left(1+\frac{\mu_{a}\lambda^{2}}{\mu\lambda_{a}^{2}}\right)+\frac{h^{2}}{k^{2}\varepsilon\mu}\left(\frac{1-\lambda^{2}}{\lambda_{a}^{2}}\right)^{2}}{\left(1+\frac{\varepsilon_{a}\lambda^{2}}{\varepsilon\lambda_{a}^{2}}\right)\left(1+\frac{\mu_{a}\lambda^{2}}{\mu\lambda_{a}^{2}}\right)-\frac{h^{2}}{k^{2}\varepsilon\mu}\left(1-\frac{\lambda^{2}}{\lambda_{a}^{2}}\right)^{2}},$$

$$Y_{-\tau}\cong\frac{\pi i}{\Gamma(\tau)\Gamma(1+\tau)}\left(\frac{\lambda_{s}r_{0}}{2}\right)^{2\tau}\frac{\left(1+\frac{\varepsilon_{a}\lambda^{2}}{\varepsilon\lambda_{a}^{2}}\right)\left(1-\frac{\mu_{a}\lambda^{2}}{\mu\lambda_{a}^{2}}\right)+\frac{h^{2}}{k^{2}\varepsilon\mu}\left(1-\frac{\lambda^{2}}{\lambda_{a}^{2}}\right)^{2}}{\left(1+\frac{\varepsilon_{a}\lambda^{2}}{\varepsilon\lambda_{a}^{2}}\right)\left(1-\frac{\mu_{a}\lambda^{2}}{\mu\lambda_{a}^{2}}\right)-\frac{h^{2}}{k^{2}\varepsilon\mu}\left(1-\frac{\lambda^{2}}{\lambda_{a}^{2}}\right)^{2}},$$

$$Y_{\tau}\cong\frac{\pi}{\Gamma(\tau)\Gamma(1+\tau)}\left(\frac{\lambda_{s}r_{0}}{2}\right)^{2\tau}\frac{2h}{k\varepsilon_{s}}\frac{1-\lambda_{s}^{2}/\lambda_{a}^{2}}{\left(1+\frac{\varepsilon_{a}\lambda_{s}^{2}}{\varepsilon_{s}\lambda_{a}^{2}}\right)\left(1-\frac{\mu_{a}\lambda_{s}^{2}}{\mu_{s}\lambda_{a}^{2}}\right)-\frac{h^{2}}{k^{2}\varepsilon_{s}\mu_{s}}\left(1-\frac{\lambda_{s}^{2}}{\lambda_{a}^{2}}\right)^{2}}$$

$$Z_{\tau}^{*}\cong\frac{\pi}{\Gamma(\tau)\Gamma(1+\tau)}\left(\frac{\lambda_{s}r_{0}}{2}\right)^{2\tau}\frac{2h}{k\mu_{s}}\frac{1-\lambda_{s}^{2}/\lambda_{a}^{2}}{\left(1+\frac{\varepsilon_{a}\lambda_{s}^{2}}{\varepsilon_{s}\lambda_{a}^{2}}\right)\left(1-\frac{\mu_{a}\lambda_{s}^{2}}{\mu_{s}\lambda_{a}^{2}}\right)-\frac{h^{2}}{k^{2}\varepsilon_{s}\mu_{s}}\left(1-\frac{\lambda_{s}^{2}}{\lambda_{a}^{2}}\right)^{2}}$$

If the direction of wave propagation is transverse to the orientation of fibers, we can put $h=0, \quad \lambda^2 = k^2 \varepsilon \mu$, then

$$Z_\tau \cong \frac{\pi i}{\Gamma(\tau)\Gamma(1+\tau)} \left(\frac{\lambda_s r_0}{2} \right)^{2\tau} \frac{1-\frac{\mu_s}{\mu_a}}{1+\frac{\mu_s}{\mu_a}},$$

$$Y_\tau \cong \frac{\pi i}{\Gamma(\tau)\Gamma(1+\tau)} \left(\frac{\lambda_s r_0}{2} \right)^{2\tau} \frac{1-\frac{\varepsilon_s}{\varepsilon_a}}{1+\frac{\varepsilon_s}{\varepsilon_a}}, \quad Z_\tau^* \cong Y_\tau \cong o \tag{17}$$

Infinite determinant is solved by the method of successive approximations. In the case of transverse wave propagation that follows from (17), functions of transverse - electric and magnetic fields $\pi^* \quad and \quad \pi$ are independent.
Infinite determinant splits into two independent determinants

$$\left\| \delta_{s\tau} + Z_\tau G_{s\tau} \right\| = 0, \qquad \left\| \delta_{s\tau} + Y_\tau G_{s\tau} \right\| = 0$$

having the same structure

$$\begin{vmatrix} 1+Z_1 G_{-1-1} & Z_0 G_{-1\,0} & Z_1 G_{-11} \\ Z_1 G_{0-1} & 1+Z_0 G_{00} & Z_1 G_{11} \\ Z_1 G_{1-1} & Z_0 G_{0\,0} & 1+Z_1 G_{11} \end{vmatrix} = 0$$

From (17), during the propagation of long transverse waves in the reinforced medium, when matrix and fillers have equal values of magnetic permeability, only then will transverse-electric field appear. In the case when the matrix and filler medium have the same dielectric permeability only then will transverse-magnetic field appear during the propagation of long transverse waves. If we consider the case where the electric field is in reinforced medium with constant wave dispersion then $h \neq 0$.

Functions of the electromagnetic field $\pi_s \quad and \quad \pi_s^*$ should be periodic for variables $x_2 \quad and \quad x_3$, so the solution of system (8) is in the form

$$V_{mn}^{(s)} = C_s e^{2\pi i d [m\cos\varphi + bn\cos(a-\varphi)]},$$

$$W_{mn}^{(s)} = S_s e^{2\pi i d [m\cos\varphi + bn\cos(a-\varphi)]} \tag{18}$$

where $V_{mn}^{(s)}$ and $W_{mn}^{(s)}$ are profiled periodic functions, if the condition is satisfied

$$d\cos\varphi = \rho, \qquad d\cos(a-\varphi),$$

$$\rho, q = 0,1,\ldots,\infty$$

When we put (18) into (8) we obtain an infinite system of algebraic equations.Then from their conditions of generality we can see infinite determinant is zero, which enables us to find the acceptable values of parameters h. In the case (10) of longitudinal wave propagations

$$G_{s\tau} = i^s e^{-is\varphi} \sum_\psi \sum_\rho {}' H_{s+\tau}(d_1 \lambda_\rho) e^{2\pi i \rho d \sin\psi - is\psi}, \tag{19}$$

where $\lambda_\rho^2 = k^2 \varepsilon_\rho \mu_\rho - h^2$.

Based on equations (10) the correlation between constants can be determined $C_s^j = g_s(h_j) G_0^j, \qquad S_s^j = g_s(h_j) S_0^j$
and unknown functions of the field are obtained through the sum of all admissible solutions

$$\pi_s = \sum_{\rho}\sum_{q}\sum_{j} e^{2\pi i(\rho m+qn)+ihjx_1}\Big\{S^{j}_{\rho q}\sum_{\tau=-\infty}^{\infty} q_\tau(h_j)\Big[\int_\tau(\lambda r_{mn}) -$$

$$- Z_\tau H_\tau(\lambda r_{mn})\ \Big]e^{i\tau v_{mn}} - S^{j}_{pq} e^{2\pi i(pm+qn)+ihjx_1}\sum_{\tau=-\infty}^{\infty} q_\tau(h_j) Y^{*}_\tau H_\tau(\lambda r_{mn}) e^{isv_{mn}}\ \Big\},$$

$$\pi^{*}_s = \sum_{\rho}\sum_{q}\sum_{j} e^{2\pi i(\rho m+gn)+ihjx_1}\Big\{S^{j}_{\rho q}\sum_{\tau=-\infty}^{\infty} q_\tau(h_j)\Big[\int_\tau(\lambda r_{mn}) -$$

$$- Y_\tau H_\tau(\lambda r_{mn})\ \Big]e^{i\tau v_{mn}} - C^{j}_{pq}\sum_{\tau=-\infty}^{\infty} q_\tau(h_j) Z^{*}_\tau(\lambda r_{mn}) e^{i\tau v_{mn}}\ \Big\}.$$

III. CONCLUSION

When the diffraction of long waves takes place then electromagnetic field changes slightly in adjacent cells, allowing the sum in (19) replaced by integrals. In this case, the electromagnetic field can be characterized by approximate values of permittivity and permeability, the value of which is determined by comparing the main characteristics of the field in homogeneous anisotropic and heterogeneous mediums. In the resulting solution, dispersion effects in the structure are taken into account, as effective parameters depend on the frequency of the field.

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